Slow light under double-double EIT regime in spherical quantum dot with hydrogen impurity

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Abstract

This work is dedicated to the study of the weak probe pulse propagation through the medium containing semiconductor quantum dots with on-center hydrogen impurity. By adding two strong cw control lasers, the four-level cascade light-matter coupling scheme can be formed, leading to the double-double electromagnetically induced transparency. The temporal profile of the output probe pulse is calculated by solving Maxwell-Bloch equations, with the help of the Fourier transform method. It is shown that the control field intensities can significantly affect the group velocity of the probe pulse, therefore creating a very efficient slow light generation mechanism. These results can be further applied to the fields of magnetometry, quantum telecommunications and quantum information processing.

Introduction

Throughout the decades, a lot of attention has been devoted to the study of the light manipulation, with various applications in quantum optics and photonics [1]. One prosperous application is slow light, obtained by reducing the group velocity of light by several orders of magnitude [2]. The technique largely used to achieve such an effect is via the electromagnetically induced transparency (EIT). This phenomenon allows the medium, previously opaque for the weak probe laser, to become transparent in the presence of another, strong control field [3].

Typically, the EIT is obtained with two lasers. However, adding another control field can lead to the formation of two transparency windows, which is called a double-double EIT [4]. In this work, this type of coupling is achieved by using the four-level cascade scheme, with the levels of the GaAs spherical quantum dot (SQD) with the on-center hydrogen impurity. Here, the charge carriers are confined in all three dimensions, and the discrete atom-like energy structure is formed. Using semiconductor SQDs improves the implementation and controllability of the experimental setup [5].

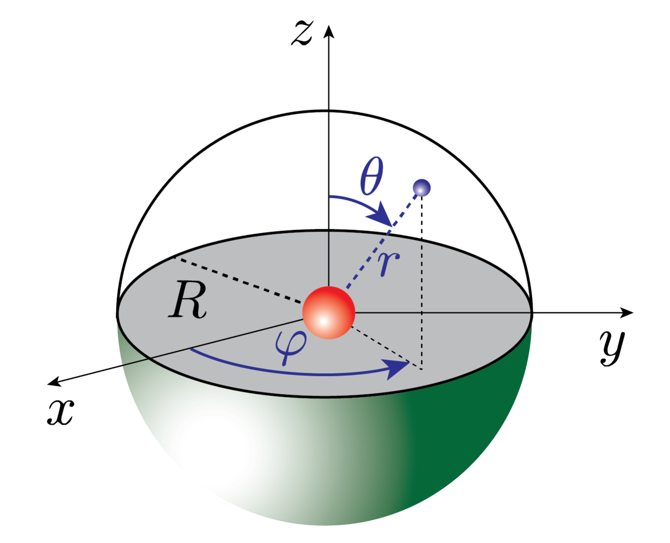

Figure 1: The schematic depiction of the SQD with the on-center hydrogen impurity.

Theory

The energies and wave functions of the SQD (Fig. 1) can be found by using the variational *Lagrange mesh method* (LMM), where the nodes of the mesh are the roots of shifted Legendre polynomials. This is done by solving the Hamiltonian eigenproblem of the dot with the radius R (in effective atomic units, i.e. for $m^* = e = \hbar = 4\pi\varepsilon_0\varepsilon_r = 1$), with expanding the wave function Ψ_m as follows:

$$H_{\text{SQD}} = -\frac{1}{2}\frac{\partial^2}{\partial r^2} + \frac{1}{2r^2}L^2 - \frac{1}{r} + V_c(r), \qquad \Psi_m(r,\theta,\varphi) = \sum_{i=1}^{N}\sum_{l=|m|}^{L} c_i f_i\left(\frac{r}{R}\right) Y_{lm}(\theta,\varphi), \tag{1}$$

where $V_c = 0$ for $r \le R$ and $V_c \to \infty$ otherwise, while $f_i(r/R)$ are Lagrange functions [6].

The light–matter interaction Hamiltonian for the four-level cascade configuration (Fig. 2 (a)) under the dipole and rotating-wave approximation in the co-rotating frame is:

$$H = \hbar\left(\Delta_p\sigma_{22} + (\Delta_p + \Delta_c)\sigma_{33} + (\Delta_p + \Delta_c + \Delta_d)\sigma_{44} - (\Omega_p\sigma_{21} + \Omega_c\sigma_{32} + \Omega_d\sigma_{43} + \text{c.c.})\right), \tag{2}$$

where $\sigma_{ij} \equiv |i\rangle\langle j|$. Here, $\Omega_s = d_{ij}E_s/(2\hbar)$ and $\Delta_s = \omega_{ij} - \omega_s$ are the Rabi frequency and detuning, respectively, with different labels s, i, j for the probe $(p, 2, 1)$, control $(c, 3, 2)$ and additional control field $(d, 4, 3)$.

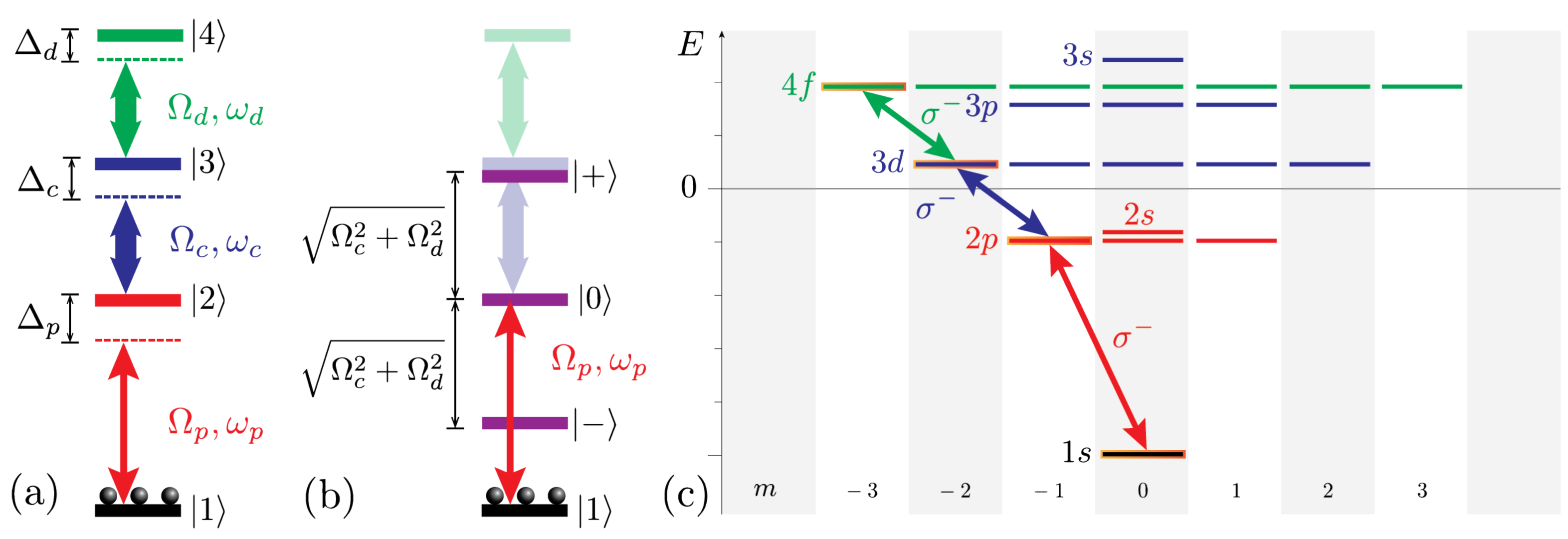

Figure 2: (a) The light–matter coupling scheme in the bare-state basis. (b) Dressed states of the SQD with $\Delta_p = \Delta_c = \Delta_d = 0$. (c) The schematic depiction of three laser fields and few lowest energy levels for the SQD with $R = 81.76\,\text{nm}$.

The dynamics of the system is governed by the combination of Liouville and propagation equation under the slowly-varying envelope approximation:

$$\frac{\partial\rho}{\partial t} = \frac{1}{i\hbar}[H,\rho] + \Lambda\rho, \qquad \left(\frac{\partial}{\partial z} + \frac{1}{c}\frac{\partial}{\partial t}\right)\Omega_p = i\frac{N_d\omega_p|d_{21}|^2}{4\varepsilon_0\hbar c}\rho_{21}, \tag{3}$$

which complete the system of *Maxwell-Bloch equations* (MBEs). Here, ρ is the density matrix, $\rho_{ij} = \langle i|\rho|j\rangle$, and the decoherence term $\Lambda\rho$ contains (phenomenological) decay rates γ_{ij}.

If the probe field is weak, the upper three levels are almost decoupled from the ground state $|1\rangle$. Under the assumption $\Delta_c + \Delta_d = 0$, two strong fields modify the other levels as:

$$\begin{aligned} |0\rangle &= \sin\theta\,|2\rangle - \cos\theta\,|4\rangle\,, \\ |+\rangle &= -\cos\theta\sin\phi\,|2\rangle + \cos\phi\,|3\rangle - \sin\theta\sin\phi\,|4\rangle\,, \qquad \tan\theta = \frac{\Omega_d}{\Omega_c}, \quad \tan 2\phi = \frac{2\Omega}{\Delta_c}, \\ |-\rangle &= \cos\theta\cos\phi\,|2\rangle + \sin\phi\,|3\rangle + \sin\theta\cos\phi\,|4\rangle\,, \end{aligned} \tag{4}$$

where $\Omega = \sqrt{\Omega_c^2 + \Omega_d^2}$. Together with $|1\rangle$, these states form the *dressed-state* basis (Fig. 2 (b)).

In the stationary regime, MBEs can be solved under the formalism of dressed states. If $\Delta_c = \Delta_d = 0$ ($\phi = \pi/4$) and $\gamma_{ij} = \gamma$, the absorption and dispersion curve ($\text{Im}\chi$ and $\text{Re}\chi$, respectively, obtained by calculating the susceptibility $\chi = N_d|d_{21}|^2\rho_{21}/(\varepsilon_0\hbar\Omega_p)$), are proportional to

$$\text{Im}\rho_{21} = \frac{1}{2}\frac{\Omega_p}{1+\frac{\Omega_d^2}{\Omega_c^2}}\frac{\gamma}{(\Delta_p+\Omega)^2+\gamma^2} + \frac{1}{2}\frac{\Omega_p}{1+\frac{\Omega_c^2}{\Omega_d^2}}\frac{\gamma}{\Delta_p^2+\gamma^2} + \frac{1}{2}\frac{\Omega_p}{1+\frac{\Omega_d^2}{\Omega_c^2}}\frac{\gamma}{(\Delta_p-\Omega)^2+\gamma^2}, \tag{5}$$

$$\text{Re}\rho_{21} = \frac{1}{2}\frac{\Omega_p}{1+\frac{\Omega_d^2}{\Omega_c^2}}\frac{\Delta_p+\Omega}{(\Delta_p+\Omega)^2+\gamma^2} + \frac{1}{2}\frac{\Omega_p}{1+\frac{\Omega_c^2}{\Omega_d^2}}\frac{\Delta_p}{\Delta_p^2+\gamma^2} + \frac{1}{2}\frac{\Omega_p}{1+\frac{\Omega_d^2}{\Omega_c^2}}\frac{\Delta_p-\Omega}{(\Delta_p-\Omega)^2+\gamma^2}, \tag{6}$$

where (5) is the sum of three Lorentzian-shaped peaks, with their height depending on Ω_c and Ω_d.

Results

The LMM with $N = 40$ and $L = 20$ is used to obtain the energy level structure for the SQD with $R = 81.76\,\text{nm}$, which is given in Fig. 2 (c) ($m^* = 0.067\,m_0$ and $\varepsilon_r = 12.9$ for GaAs).

MBEs are solved by using the Fourier transform method (Fig. 3), with the initial conditions $\rho_{11}(z,0) = 1$ and the remaining $\rho_{ij}(z,0) = 0$, while the initial Gaussian-shaped probe pulse is $E_p(0,t) = E_{p0}\exp\left(-w^2(t-t_0)^2\right)$. The other parameters are: $E_{p0} = 250\,\text{V/m}$, $w = 20\,\text{GHz}$, $\Delta_p = 10^{11}\,\text{Hz}$, $\Delta_c = \Delta_d = 0$ and $\gamma = 10\,\text{GHz}$. The number density of SQDs is $N_d = 10^{21}\,\text{m}^{-3}$ and sample length is $D = 200\,\mu\text{m}$. The results are explained by analyzing the absorption and dispersion curve in the stationary regime (Fig. 4).

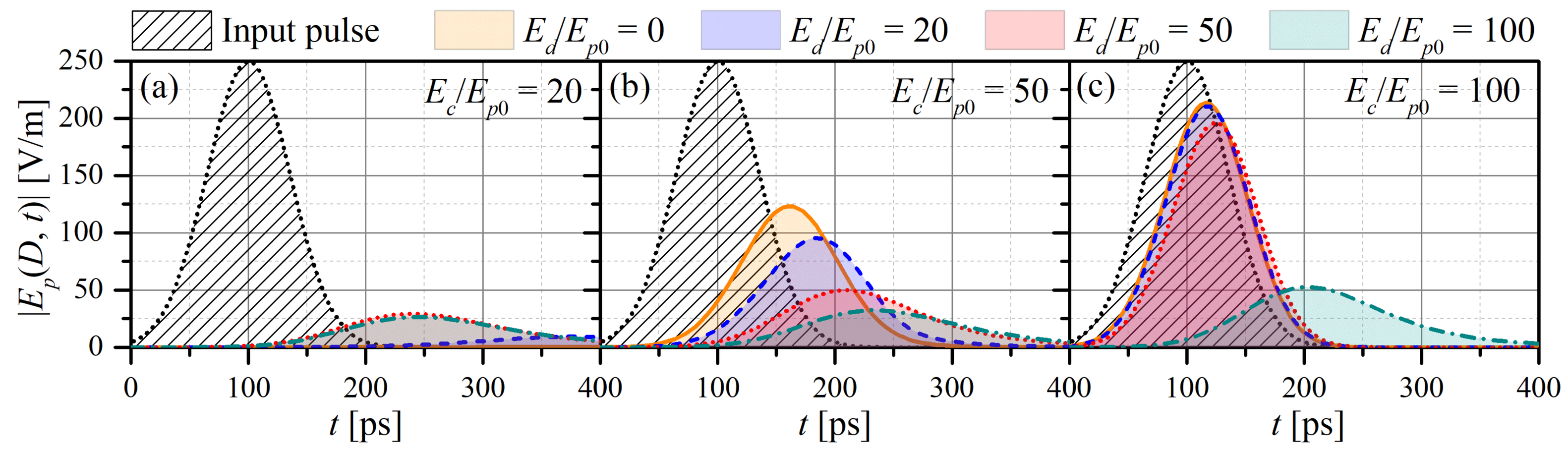

Figure 3: The temporal profile of the output probe pulse envelope for different values of the strength of the control fields.

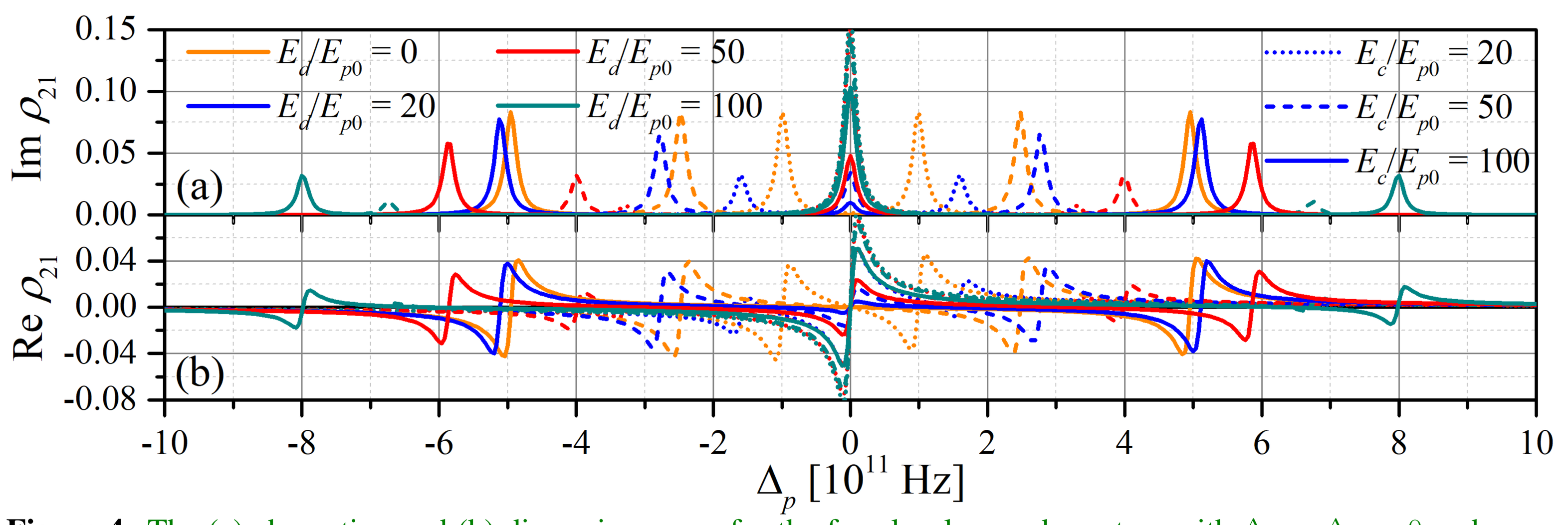

Figure 4: The (a) absorption and (b) dispersion curve for the four-level cascade system with $\Delta_c = \Delta_d = 0$ and several values of the strength of the control fields.

The group index $n_g = c/v_g$, relative pulse width $\delta = \sigma_{\text{out}}/\sigma_{\text{in}}$, efficiency $\eta = W_{\text{out}}/W_{\text{in}}$ and fidelity $\xi = |S|/W_{\text{in}}$ of the output pulse (Fig. 5) are calculated by using the expressions ($z_{\text{in}} = 0$, $z_{\text{out}} = D$):

$$\langle t^k\rangle_{\text{in,out}} = \int_0^\infty t^k|E_p(z_{\text{in,out}},t)|^2\,dt, \qquad S = \int_0^\infty E_p^*\left(0, t - \frac{D}{v_g}\right)E_p(D,t)\,dt, \tag{7}$$

from which we have $v_g = D/(\langle t\rangle - t_0)$, $\sigma_{\text{in,out}} = \sqrt{\langle t^2\rangle_{\text{in,out}} - \langle t\rangle^2_{\text{in,out}}}$ and $W_{\text{in,out}} = \langle t^0\rangle_{\text{in,out}}$.

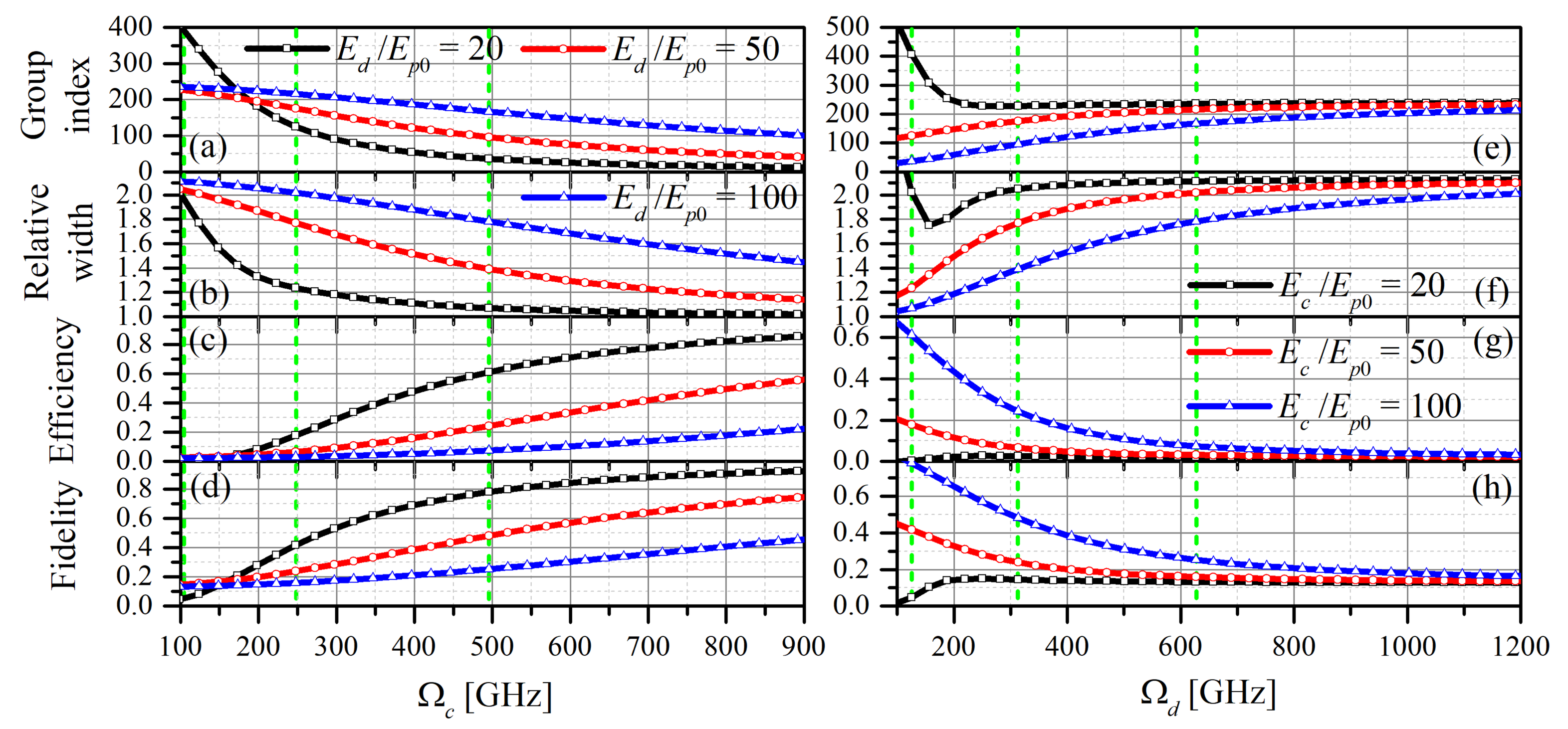

Figure 5: The output pulse parameters as functions of the Rabi frequency of the (a–d) control field and (e–h) additional control field. The values of Ω_c and Ω_d used in the previous figures are given in green dashed lines.

Conclusions

- As a consequence of the EIT effect in the three-level cascade scheme, the probe light in GaAs SQDs can be slowed down a few dozen times, as shown in the previous work [6].
- Applying the additional control field results in the formation of the middle absorption peak and two transparency windows instead of one, with the peak position and height depending on Ω_c and Ω_d.
- The connection between (5) and (6) implies that the realization of slow light (for probe detunings where the dispersion curve has a negative slope) is possible in the double-double EIT regime.
- From Fig. 5 (e) it follows that, for large E_c/E_{p0}, n_g increases with the increase of Ω_d – the additional control field can reduce v_g by one order of magnitude comparing to the three-level case.
- Finding the right balance between n_g and η is of a great importance for future applications of semiconductor quantum dots as optical switches and optical buffers.

References

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