Quantum Droplets in Dipolar Ring-shaped Bose-Einstein Condensates

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Abstract

We study the formation of quantum droplets in dipolar Bose-Einstein condensates in a ring-shaped geometry using numerical techniques. A condensate is initially prepared in a stable ground state of the system, and droplet formation is triggered by a sudden quench of the contact interaction. We investigate how the number of the obtained droplets depend on the total number of atoms in the system, as well as on the strength of the contact and the dipole-dipole interaction. These results can be used in experiments to fine-tune parameters of the system in order to produce droplets of desired size. Furthermore, we study the emergence of supersolidity in the system, when droplets are formed due to the contact interaction quench, but the common phase is still preserved among spatially separated droplets. The quasi-1D geometry imposes additional constraints in the system, in particular when the particle density is higher, such that quantum fluctuation effects become more prominent. We use the Bogoliubov-Popov theory for dipolar Bose systems, including the dipolar analogue of the Lee-Huang-Yang correction, and take into account the condensate depletion due to quantum fluctuations.

Introduction

- Dipolar Bose-Einstein condensates
  - Particles with permanent magnetic or electric dipole moment
  - Experimental realization in 2005 in ^{52}Cr [1], later in ^{164}Dy, ^{168}Er, ...

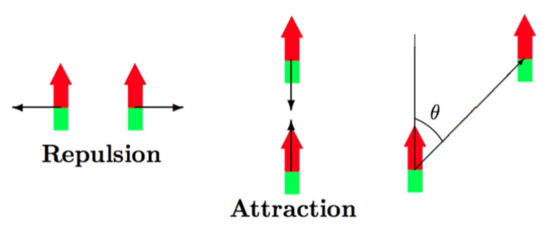


Fig. 1. Schematic illustration of the sign of the DDI for different relative position of dipoles

- There are two competing interactions
  - Contact interaction: short-ranged, isotropic, proportional to the s-wave scattering length a_s

$$V_c(\mathbf{r}-\mathbf{r}') = g\delta(\mathbf{r}-\mathbf{r}'), \quad g = \frac{4\pi\hbar^2 \boldsymbol{a_s}}{m}$$

  - Dipole-dipole interaction (DDI): long-ranged, anisotropic, proportional to the dipolar length $a_{\rm dd}$

$$V_{\rm dd}(\mathbf{r}-\mathbf{r}') = \frac{3\hbar^2 a_{\rm dd}}{m}\frac{1-3\cos^2\theta}{|\mathbf{r}-\mathbf{r}'|^3}, \quad \boldsymbol{a_{\rm dd}} = \frac{\mu_0\mu^2 m}{12\pi\hbar^2}$$

- The existence of Feshbach resonances allow the tuning of the s-wave scattering length to arbitrary value by changing the strength of the magnetic field, thus making the dipolar effects more prominent

Quantum droplets

- A system of confined dipoles can be stable only if a strong enough repulsive contact interaction is present, to resist the collapse of the system due to the attractive part of the DDI. Quantum fluctuations are usually considered negligible at low temperatures. However, in some cases, a system that is expected to collapse according to the mean-field description, can be stabilized due to quantum fluctuations, that effectively act as an additional repulsive interaction. Therefore, instead of collapsing the system transition to a new state called a quantum droplet.
- Quantum droplets are self-bound states, with densities at least one order of magnitude higher than the BEC
- First experimental realization of quantum droplets in 2016 [2]

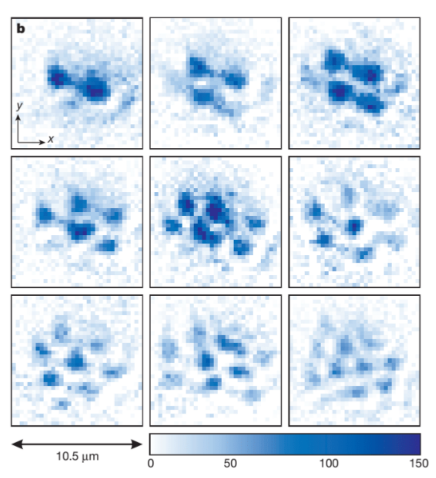


Fig. 2. Growth of a microscopic droplet crystal. Adapted from [2].

Theory

- Mean-field theory is not sufficient to describe the emergence of quantum droplets
- Beyond-mean-field corrections due to quantum fluctuations: derived in 1957 for the contact interaction (LHY correction) [3], and in 2012 for the DDI [4]
- The extended Gross-Pitaevskii equation (eGPE) in local-density approximation

$$i\hbar\frac{\partial\Psi(\mathbf{r},t)}{\partial t} = \left[-\frac{\hbar^2}{2m}\Delta + U_{\rm trap}(\mathbf{r}) + gn(\mathbf{r},t) + \int d\mathbf{r}'\, V_{\rm dd}(\mathbf{r}-\mathbf{r}')n(\mathbf{r}',t) + \Delta\mu(\mathbf{r},t)\right]\Psi(\mathbf{r},t),$$

  Here, $n(\mathbf{r},t)$ is the total density of particles, $n(\mathbf{r},t) = |\Psi(\mathbf{r},t)|^2 + \triangle n(\mathbf{r},t)$, where $\Psi(\mathbf{r},t)$ is the condensate wave function, and $\triangle n(\mathbf{r},t)$ is the condensate depletion, and the normalization condition holds: $\int d\mathbf{r}\, n(\mathbf{r},t) = N_a$.
- The corrections to the condensate density and chemical potential are given by

$$\Delta n(\mathbf{r},t) = \frac{8}{3}|\Psi(\mathbf{r},t)|^3\sqrt{\frac{a_s^3}{\pi}}Q_3(\epsilon_{\rm dd}),$$
$$\Delta\mu(\mathbf{r},t) = \frac{32}{3}g\,|\Psi(\mathbf{r},t)|^3\sqrt{\frac{a_s^3}{\pi}}Q_5(\epsilon_{\rm dd}),$$

  where $\epsilon_{\rm dd} = a_{\rm dd}/a_s$, and $Q_l(x) = \int_0^1 du\,(1-x+3xu^2)^{l/2}$
- The system is confined in a ring-shaped geometry

$$U_{\rm trap}(\mathbf{r}) = \frac{1}{2}m\left[\omega_\rho^2(\rho-R)^2 + \omega_z^2 z^2\right],$$

  where $\rho = \sqrt{x^2+y^2}$ is the radial distance, R is the radius of the ring, and ω_ρ and ω_z are the corresponding trap frequencies. The dipoles are oriented along the z axis.
- The eGPE is numerically solved using the split-step semi-implicit Crank-Nicolson method [5]

Results: Condensate depletion

- We model the experimental protocol for obtaining quantum droplets:
  - The system is prepared in a stable BEC ground state
  - Rapid quench of the contact interaction strength is performed at $t = 0$
- Parameters: ^{164}Dy atoms, $a_{\rm dd} = 132\,a_0$, $a_s^{(0)} = 132\,a_0$, $R = 10\mu m$, $\omega_\rho = \omega_z = 2\pi\times 600$Hz

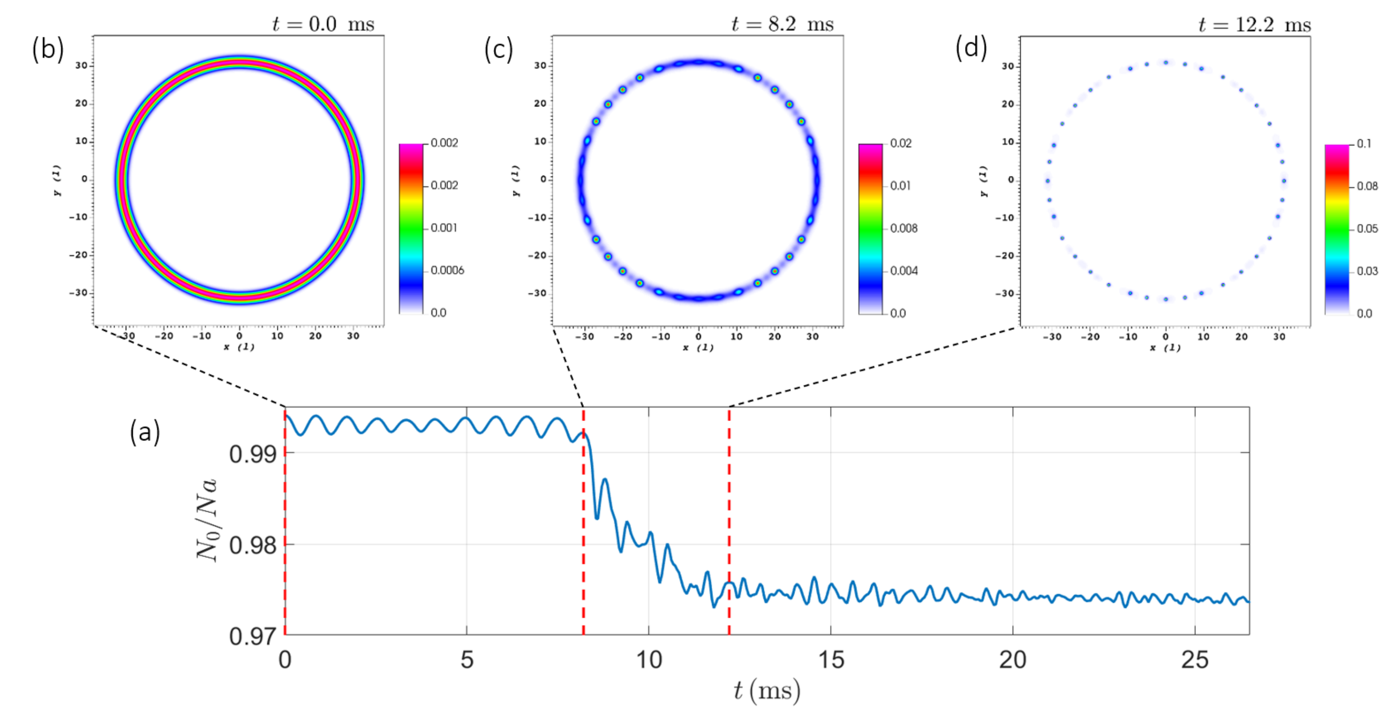


Fig. 3. (a) Time dependence of the condensate fraction after the contact interaction quench $a_s = 132\,a_0 \longrightarrow 45\,a_0$, for $N_a = 10^4$ atoms. (b)-(d) Two-dimensional density distributions in the xy plane at characteristic times, illustrating the emergence of quantum droplets. The transition of the system to the droplet phase corresponds to the sudden decrease in the condensate fraction by 2%, that is, to the increase in quantum fluctuations.

Results: Phase diagram and supersolid states

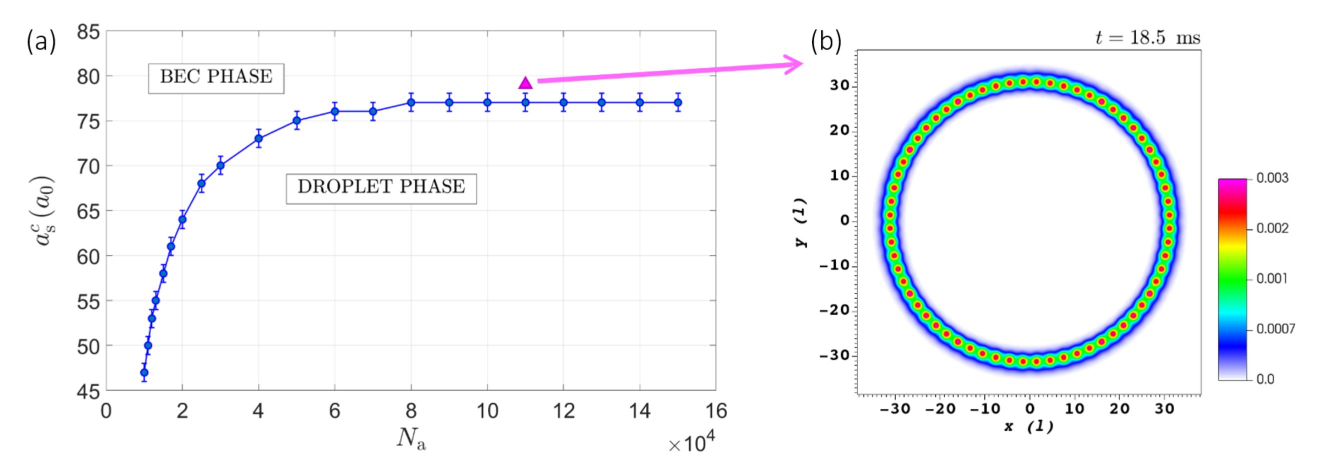
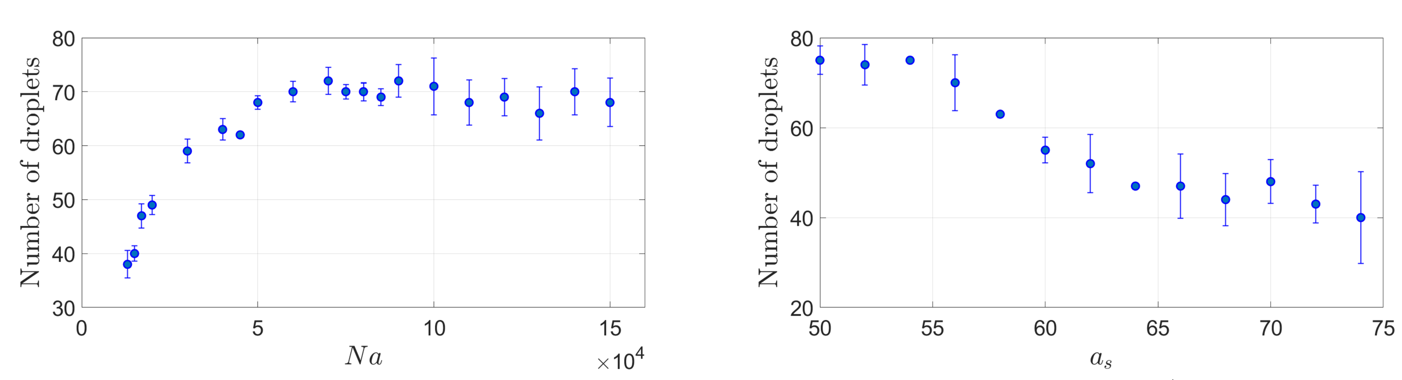


Fig. 4. (a) Phase diagram in the plane (N_a, a_s^c) with denoted regions corresponding to a BEC and a droplet phase, after the contact interaction quench. It is important to note that the line does not represent a clear separation between phases. In the narrow region close to the boundary the system can be characterized as a supersolid, where quantum droplets are immersed into a superfluid background. (b) An example of the supoersolid state (2D density distribution). Parameters: $N_a = 1.1\cdot 10^5$ atoms, contact interaction quench $a_s : 132\,a_0 \longrightarrow 79\,a_0$.

Results: Number of droplets



(a) $a_s = 132\,a_0 \longrightarrow 55\,a_0$ (b) $N_a = 6\cdot 10^4$

Fig. 5. Dependence of the number of obtained droplets: a) on the total atom number, for fixed contact interaction quench; (b) on the scattering length to which the system is quenched from $a_s^{(0)} = 132\,a_0$, for fixed atom number.

Conclusions and future work

- Quantum fluctuations are the main effect that stabilizes the system, by expelling particles from the condensate into some excited states, and thus enabling the emergence of quantum droplets
- Conditions for droplet formation in the selected protocol, which can be seen from the phase diagram:
  - Sufficiently large contact interaction quench
  - Sufficiently high condensate density
- Study of the number of formed droplets depending on the total atom number and the contact interaction quench
- Further study: the emergence and properties of supersolid states

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